Mathematics Teacher

DEVOTED TO THE INTERESTS OF MATHEMATICS
IN JUNIOR AND SENIOR HIGH SCHOOLS

VOLUME XVIII NOVEMBER, 1925 NUMBER 7

Published by the
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
NEW YORK YONKERS, N. Y.

Entered as second-class matter, November 10, 1921, at the Post Office at Yonkers, N. Y., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized November 17, 1921.

THE MA[illegible]MATICS TEACHER

[illegible] JOURNA[illegible] NATIONAL [illegible]UNCIL OF TEACHERS
[illegible] MATHEMATICS

[illegible]ization has for its object the advancement of mathematics [illegible] junior and senior high schools. All persons interested in [illegible] and mathematics teaching are eligible to membership. [illegible] receive the official journal of the National Council—[illegible] TEACHER—which appears monthly, except June, July, [illegible] September.

[illegible]ce relating [illegible] matters, subscriptions, advertisement, and other b[illegible] should be addressed to

JOHN [illegible], Editor-in-Chief
The L[illegible] Teachers College
[illegible] W[illegible] Street, New [illegible] City

[illegible]RIPTION [illegible] PER YEAR (eight numbers)
[illegible]ge, 50 cents [illegible] year; Canadian postage, 25 cents per year. If remittance is made by check, five cents should be added for exchange.

THE MATHEMATICS TEACHER

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OBJECTIVES IN THE TEACHING OF MATHEMATICS

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The first step in making a modern course of study in mathematics is to set up a list of desirable objectives. The second step is to determine the nature and the extent of the subject-matter which will best enable teachers to realize these objectives. The third step is to develop the best methods of teaching the selected subject matter. The fourth step is to organize a testing program which can be used by teachers to see whether the objectives have been attained. Corollary to the last two steps is the recognition of the need for an analysis of how pupils learn most efficiently and easily. It is the purpose of this paper to discuss only the first step mentioned above, namely, the setting up of a desirable list of objectives. It is hoped that attention may later be given to the remaining steps.

It is evident that no satisfactory list of objectives can be set up by consulting only existing courses of study. While they may be used as evidence of what is being taught throughout the country they do not often give much help in suggesting what ought to be taught. Moreover, it is well known that some of these courses, if not all, tend to perpetuate certain obsolete processes and antiquated business methods.

It is equally true that the best list of objectives cannot be secured by making an inventory of only the current textbooks in mathematics. They, too, are frequently guilty of overemphasizing unimportant or obsolete material. It is also true that textbook writers are not always able to suggest newer and better things.

The standardized test makers of recent years have erred in including in their tests exercises and problems that thoughtful teachers everywhere have no desire to see perpetuated in our schools. In fact many of these undesirable elements were obtained by the makers of tests from existing courses of study and text books. Thus, it is obvious that such tests cannot be used as the sole basis for determining a list of desirable objectives.

We also know that it is not safe to try to determine what mathematics ought to be taught by counting the frequency with which certain mathematical terms are used in a few current editions of newspapers and magazines.

Finally, it is fair to say that we cannot determine our objectives by going out in the world and asking Tom, Dick and Harry what mathematics is useful to them. The fact is that not one of them ever knows just what use he has made of mathematics. Moreover, they probably have given no thought to the question of determining how they might have used mathematics profitably if they had only thought about it a little while.

Any and all of the above criteria may be of service to us in making our list of desirable objectives, but they will not suffice. If the objectives set up are to meet our modern needs, we must have at least one other criteria that is supported by vision though it should not be visionary. This last criteria is the opinion of expert teachers of mathematics—those who are able not only to tell how they use mathematics, but also to show how mathematics may be used in the present and in the future for the betterment of mankind.

When people have to consult with experts they usually get the opinion of the best experts they can find. In preparing the lists of objectives that are given in this paper I had the assistance of a large number of experienced teachers of mathematics who are students at Teachers College and a few more who are considered experts in the field of mathematics.

The lists given here are offered in the hope that we may secure the constructive criticisms of other experts who thus far have had no chance to co-operate in the selection of these objectives.

The remainder of the paper I have divided into five parts. The first part contains a list of general objectives which has two subdivisions, namely, one on *general objectives* which may be desirable in any field and another on *strictly mathematical objectives.* The second part is composed of a list of objectives in the field of algebra, particularly the more general ones pertaining to desirable attitudes of mind, to appreciations, and to ideals. The third part is confined to a list of *specific objectives* in the teaching of algebra and has to do entirely with the abilities to be developed in the ninth grade.

The fourth part is a suggestive list of desirable formulas to be used in connection with the accompanying objectives.

The fifth part is a *selected bibliography on objectives in the teaching of mathematics.*

Other objectives of a specific nature need to be set up for the other subjects such as geometry, trigonometry and so on, but it is not the purpose of this paper to discuss them. The end of the discussion of desirable objectives is not in sight, but it is hoped that we have made some progress in setting forth a method of attack and that other teachers may help by offering further suggestions.

PART I

OBJECTIVES IN THE TEACHING OF MATHEMATICS

I. General Objectives

1. To give exercise in and appreciation of the fundamental mode of thought which mathematical thinking best represents. This is done by establishing and extending ideals of

   a. Simplicity in language.

      Mathematics is generally used to give clear and simple formulation to a law after it has been discovered.

   b. Accuracy in reasoning.

      The inevitable certainty of conclusions is more clearly brought out in mathematics than in any field of knowledge. Moreover, in no other field of knowledge is inaccuracy in reasoning more quickly discovered or vagueness in statement more easily detected.

   c. Originality in thought.

      The solution of a difficult problem calls for the best that is in a pupil and should make a strong appeal to his desire for self expression.

   d. Reliable information.

      Each pupil who has the ability to learn mathematics should feel it a duty to study it as he should the other great branches of learning e. g., Science, letters, the fine arts, religion and philosophy. He may not be expected to know mathematics as a tool in every case, but he should be able to use it as a basis for evaluating the work of his fellows.

2. To establish, organize and make cooperative certain important and specific habits:

a. Of action, *e. g.*, cleanliness, neatness, and uniformity.

b. Of thinking, *e. g.*, to afford an opportunity for concentration and constructive imagination.

c. Of moral conduct, *e. g.*, to give an opportunity for the extension of matters of self-confidence, self-reliance, and honesty.

d. Of character, e. g., to assist in establishing and extending ideals of reverence for truth, beauty and the like.

II. Strictly Mathematical Objectives

1. To increase and extend the power

a. Of understanding and of analyzing relations of quantity and space which are necessary to an appreciation of the progress of civilization and of deducing from them proper conclusions.

This is fundamental especially in all scientific and economic fields. It involves the ability to select the essential and eliminate the non-essential from a mass of facts, to put them together in logical form, and to present them, if necessary, in a comprehensive, clear-cut, and convincing manner.

b. Of reading, interpreting, organizing, and expressing statistical and graphical data so as to meet some need or purpose.

c. Of recognizing, understanding, and using symbolic notation.

This is fundamental in giving a proper ideal of perfection in the form and effectiveness in expression. In its role as the interpreter of other fields mathematics plays a large part.

2. To extend and refine the power gained in applying mathematics so as to furnish a basis for an an interest in future mathematical work.

This is necessary to meet effectively
(*a*) The ordinary demands of daily life.
(*b*) The demands of future mathematical work.
(*c*) The demands of the work in related fields of knowledge.
The highly organized fields of business are daily becoming more dependent upon mathematics. If mathematics were taken away from us civilization would shortly collapse.

3. To furnish and increase the incentive of studying mathematics for the love of the subject.

This is one worthy use of leisure that can be justified for those who love mathematics.

PART II

APPRECIATIONS, ATTITUDES OF MIND, AND IDEALS TO BE SECURED IN THE TEACHING OF ALGEBRA

I. Appreciations

To develop an appreciation of:

1. The progress of algebra. This involves an appreciation of the historical side of algebra and its contribution to the progress of civilization.
2. The power of algebraic symbolism as shorthand.
3. Algebra as related to the other branches of mathematics. For example, to arithmetic.
4. The relation of algebra to closely related subjects such as surveying, physics, engineering, navigation, astronomy and the like.
5. The relation of algebraic formulas to general truths. For example, when a law is discovered then a statement of that law can be translated into a formula.
6. The fact that the truths of algebra are eternal.
7. The universality of functional relationships.
8. The need for expressing the data of applied problems in systematized form.
9. The value of carefully scrutinizing data in the solution of applied problems.
10. The value of careful organization of data as an aid to memory and understanding.
11. The value of carefully setting up the hypotheses in solving problems.
12. The value of algebra per se regardless of any contingent need for it in preparation for advancement in school or in college.

II. Attitudes of Mind

To develop an attitude of:

1. Feeling a responsibility for the correctness of a result; thus developing self-confidence.

2. Realizing that no problem should be considered solved without previously making a commonsense estimate of the probable result.
3. Looking for applications of algebra.
4. Being interested in and desiring to grow in the necessary algebraic skills.
5. Rejecting irrelevant data in a problem, and recognizing the need of certain missing data.
6. Wanting to discover a general rule or law when observing particular cases. This is what has been called the scientific method of observation, experimentation, and calculation.
7. Being dissatisfied with incomplete or vague results.
8. Being satisfied only when work is thorough and clear.

9 ~~10.~~ Analyzing all statements and discriminating between the true and the false; thus creating ideals of self-reliance, and devotion to truth.

III. Ideals

To create and develop ideals of:

1. Originality in the solution of problems.
2. Accuracy, thoroughness, and clearness in thinking in oral and in written expression.
3. Neatness.

PART III

The following list of objectives constitute what is thought to be desirable for the ninth grade course in algebra. In formulating this list about 200 experienced teachers have cooperated. No attempt has been made to list these objectives in order of their importance, or to make the list exhaustive. It is believed that it is a selection upon which teachers can base their discussions in trying to make a modern course of study in algebra.

OBJECTIVES IN TEACHING THE FORMULA

To develop the following abilities:

1. **To develop certain rules of mathematics and to translate them into formulas.**

This means that pupils should understand the meaning of the formula as a shorthand rule of mathematics. The rule should grow out of his experience if possible. At any rate he should be told what the formula means as far as possible. Here is where algebra begins.

2. To translate certain formulas into rules of mathematics.

This means that pupils must know how to use a formula when the need arises. That is, in getting certain required results he must be taught how to decide which is the proper formula to use.

3. To evaluate certain formulas; that is, to find the values of certain letters when the values of the others are known.

These formulas should be of a difficulty no greater than that found in the operations which the pupils have been taught or which they may be expected to understand.

Note: A list of desirable formulas for use in the ninth grade work in algebra is given in Part IV.

4. To derive one formula from another.

This means that the pupil must be able to solve a formula for one letter in terms of the other letters in that formula. This involves the ability to solve equations by means of which solution the "subject of the formula" is changed.

The types of equations involved are as follows:

(1) $2x = 6$. (2) $x + 5 = 8$. (3) $x - 4 = 7$. (4) $\frac{1}{2}x = 9$.

The solution of such equations implies a knowledge of how to use the four fundamental operations.

5. To represent by a graph certain formulas of a type no more difficult than $F = 9/5C + 32$.

This involves the ability to make a table of values for a formula.

6. To understand the idea of the dependence of one quantity upon another.

This involves the ability to appreciate the idea of one variable as a function of another.

OBJECTIVES IN TEACHING EQUATIONS OF THE FIRST DEGREE IN ONE UNKNOWN

To develop the following abilities:

1. To translate into equation form certain conditions stated in words. This involves two abilities:

(1) To give the answer to such questions as this: "What is the cost of 8 pencils at k cents each?" Answer: $C = 8k$.

(2) To write the correct equation for problems similar to this: "Twice a number increased by five is equal to twelve. Find the number." Answer: $2n + 5 = 12$.

2. To translate into words certain simple types of equations.

This is the reverse of the preceding objective.

3. To solve, algebraically, equations of the type $y = ax + b$.

4. To understand the graph of the equation $y = ax + b$ as the graph of a linear equation and as a method of solving the family of equations when y is given any particular value.

5. To use equations in solving such applied problems of algebra as are of real use in business or in science.

6. To distinguish clearly between an identity and an equation of condition.

   This means the ability to appreciate the equation of condition as "the interrogative sentence of algebra" and the identical equality as "the declarative sentence of algebra."

7. To solve simple equations containing common or decimal fractions.

   This involves the ability to solve equations of a type no harder than those which are needed to understand how to solve for a letter in any of the formulas which contain fractions.

OBJECTIVES IN TEACHING STATISTICAL GRAPHS

To develop the following abilities:

1. To interpret pictorial graphs of various kinds.

   This does not necessarily mean the ability to draw such graphs.

2. To interpret and to draw

   (1) Bar diagrams; (2) Broken line graphs; (3) Curve line graphs; and (4) Simple circular graphs.

3. To criticize the appropriateness and correctness of the use of any of the above graphs in a practical case. For example, to understand how pay envelopes are made up from graphic charts, but not to draw such charts.

4. To choose and be able to draw the appropriate graph to represent the facts in certain simple cases of statistical data.

   This implies the ability to use a table of values.

5. To make a comparison of two or more statistical graphs on the same piece of paper using the same coordinate axes.

6. To locate points using the conventional x and y coordinate axes.

   This involves the ability to understand the relation between a point and the number pair (x, y).

7. To understand, appreciate, and use directed numbers in graphic work.

OBJECTIVES IN TEACHING THE GRAPH OF A MATHEMATICAL LAW

To develop the following abilities:

1. To represent points graphically by means of the conventional coordinate system.

   In addition to an apprecaition of the relation of a point to the number pair (x, y), as already mentioned, this involves an understanding of the four quadrants and the use of a table of corresponding values for two variables.

2. To understand and use directed numbers.

3. To understand and represent, graphically, variation of the types $y = kx$ (direct) and $xy = k$ (inverse), and to show how functional relationships are made more apparent by the graph than by the equation.

4. As already stated in the preceding list of objectives, to compare graphs drawn with the same pair of coordinate axes.

5. To interpolate and exterpolate.

   This involves the use of the graph to find intermediate values (interpolate) and to extend graphs (exterpolate) for purposes of prediction.

6. To interpret the intersections of certain graphs like those used by the railroads in arranging their time tables.

7. To interpret the intersections of certain graphs as points whose coordinates represent real roots of the corresponding equations, and their non-intersection as indicating unreal or imaginary roots.

8. To understand the relation between the variables as expressed by certain special graphs which represent the equations.

   $y = x^2$, $y = x^3$, $y = e^x$, and so on.

   Some teachers are doubtful about including $y = e^x$, but most of them seem to think it should be included.

9. To construct the graph of the equation $y = ax + b$ and to explain its meaning.

   The value of x when $y = o$ should not receive undue emphasis.

10. To construct the graph of the function $y = ax^2 + bx + c$ and be able to show how it represents a family of values, the case in which $y = o$ being only a special case which is often overemphasized in the teaching of quadratic equations.

11. To construct the graphs of two simultaneous equations of the type

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

and to explain how the graphs show whether the equations are solvable.

12. To use graphs as a rough check on algebraic operations.
13. To read values from a graph quickly and accurately.
14. To construct a graph for the purpose of solving practical problems.

    This is similar to what is referred to in 6 above.

15. To use the graph in such allied fields as general science, social science, and the like.
16. To understand the elementary facts concerning a frequency curve.
17. To distinguish between the significance of the statistical graph and the graph of a mathematical law.
18. To appreciate the fact that most of the laws in mathematics can be represented graphically.

OBJECTIVES IN TEACHING DIRECTED NUMBERS

To develop the following abilities:

1. To understand and interpret the meaning of directed numbers and to represent them on an algebraic scale, thus extending the notion of number.
2. To use directed numbers in a practical way.
3. To add two directed numbers in a column or horizontally.
4. To subtract one directed number from another in a column or horizontally.
5. To combine directed numbers by addition and subtraction in a column or horizontally.
6. To multiply one directed number by another in a column or horizontally.
7. To divide one directed number by another, the operation being taught as the inverse of multiplication.
8. To understand the connection between directed numbers and the coordinate system.

9. To understand the use of directed numbers in a few simple cases taken from physics or mechanics.
10. To understand what is meant by the "absolute value" of a number.
11. To appreciate the use of directed numbers in connection with formulas, thus extending their use.
12. To distinguish between plus and minus signs as signs of quality and signs of operation.
13. To remove not more than two sets of parentheses where directed numbers are involved, the case being treated as a convenient way of indicating addition or subtraction.

OBJECTIVES IN TEACHING THE FUNDAMENTAL OPERATIONS ON ALGEBRAIC POLYMONIALS

To develop the following abilities:

1. To add, subtract, multiply, and divide in the manner indicated for directed numbers. (Objectives 3-7.)
2. To multiply a binomial by a monomial with or without using parentheses and to represent the product by means of a rectangle. (Geometric representation.)
3. To multiply any polynomial by a monomial as in 2 above.
4. To multiply a binomial by a binomial and represent the product by means of a rectangle.
5. To perform the multiplication in No. 4 above by inspection, the case not being separated into types at first, but being looked upon simply as one involving two formulas.
6. To divide a binomial by a monomial, including both integral and fractional forms.
7. To divide a polynomial by a monomial as in the preceding case.
8. To understand the ordinary symbols of aggregation, not over two "nests of parentheses" being involved.
9. To divide one simple polynomial by another simple one.

   Since the case has no important applications in elementary algebra, it should be briefly treated.
10. To understand that factoring is important in transforming one formula into another which is more easily evaluated.

11. To understand that factoring is the inverse of multiplication. For example, $10x^2 + 19x + 6 = (2x + 3)\ (5x + 2)$ because $(2x + 3)\ (5x + 2) = 10x^2 + 19x + 6$. In other words the identity is reversed.

12. To factor by taking out a common monomial factor.

13. To factor the general quadratic trinomial $ax^2 + bx + c$.

14. To factor the difference of two squares. If desired, this may be treated as a special case of the general quadratic trinomial. The so-called "type products" and "cases" of factoring should not be overemphasized. We really need only two or three types of factoring and these can be handled simply by reversing the process of multiplication.

    Teachers seem to be pretty generally agreed that factoring has been overdone and that even yet we are still expecting pupils to factor quadratics that are too difficult. Unless the pupils are to study the solution of quadratic equations by the method of factoring there seems to be no good reason for much factoring beyond that which is necessary to make desirable changes in a formula.

15. To understand why division by zero is not permitted.

    This is involved in checking certain problems in long division where the denominator may turn out to be zero.

16. To look at the first result in factoring to see if the expression cannot be factored again.

17. To understand how to bring simple exercises under the four laws of exponents as follows: $a^m \cdot a^n = a^{m+n}, (a^m)^n = a^{mn}$, $\dfrac{a^m}{a^n} = a^{m-n}$ and $(ab)^n = a^nb^m$

18. To understand the meaning given to zero, negative, and fractional exponents and to solve simple exercises where these are involved.

19. To understand that the fundamental operations are only a means of reaching some kind of result, mere skill in manipulation not being the purpose of the work. Only the types needed later, either in life problems, or in other parts of algebra, should be taught. Their relation to the work on the formula should be explained.

20. To check solutions and thus to realize when a topic has been fairly mastered.

21. To understand further the extended use of symbolism in algebra as set over against simple operations in arithmetic.

OBJECTIVES IN THE TEACHING OF FRACTIONS

To develop the following abilities:

1. To understand a fraction as an indicated division.
2. To understand the underlying principle in connection with the change of signs in the terms of a fraction. For example, that

$$\frac{a}{b}=\frac{-a}{-b}=-\frac{-a}{b}=-\frac{a}{-b}$$

3. To reduce a fraction to lower terms, and, in a few cases to higher terms.
4. To reduce an improper fraction to a mixed expression or to an integral expression.
5. To reduce a mixed expression to an improper fraction.
6. To reduce fractions to equivalent fractions having the L.C.D. when the L.C.D. can be found by inspection.
7. To add and subtract fractions having the same denominator and having different denominators when the L.C.D. can be found by inspection and where the denominators are either monomials, or else polynomials of the three ordinary factoring types, i. e., those which are the product of two binomials.
8. To multiply and divide fractions of a simple type.
9. To simplify complex fractions which are not more complicated than those which would occur in the pupils' work with formulas, or in checking fractional equations where the root is a fraction.
10. To see the connection between arithmetic and algebraic fractions.
11. To check all results.

OBJECTIVES IN TEACHING FRACTIONAL EQUATIONS

To develop the following abilities:

1. To clear an equation of fractions, the denominators being arithmetic or literal numbers, the work being limited to cases where the L.C.D. can be found by inspection.
2. To solve numerical equations containing common or decimal fractions of a simple type.

3. To solve equations containing fractions with binomial denominators.
4. To solve interest problems using the formulas
$$i = prt \text{ and } A = pr\ (1 + rt).$$
5. To solve applied problems leading to fractional equations.
6. To derive one formula from another, that is, to solve literal equations.
7. To understand why a literal equation may be considered as representing a family of problems.
8. To evaluate formulas involving fractions.
9. To check all results.

OBJECTIVES IN TEACHING RATIO, PROPORTION, AND VARIATION

To develop the following abilities:

1. To consider ratio as an abstract number, it being the quotient of one number divided by another of the same denomination; that is, feet by feet, cents by cents, and so on.
2. To understand a proportion as the equality of two or more ratios or as a fractional equation.
3. To understand that a ratio, particularly when expressed in fractional form, is subject to the various laws of fractions.
4. To understand that a proportion is subject to the various treatments given a fractional equation.
5. To understand direct variation whether considered graphically, (i.e., to interpret the graph), or symbolically, (i. e., as $y = kx$ or $y/x = k$.)
6. To understand inverse variation, whether considered graphically or symbolically, (i.e. as $xy = k$).
7. To solve simple and genuine applied problems in direct and inverse variation.
8. To understand more fully the function idea.

OBJECTIVES IN TEACHING NUMERICAL TRIGONOMETRY

To develop the following abilities:

1. To draw to scale.
2. To understand and use the metric system of measurement.

3. To understand the meaning of tangent, sine, and cosine.
4. To use the tangent, sine, and cosine in problems involving right triangles. This may involve using them either as multipliers or, less commonly, as divisors.
5. To understand the difference between direct and indirect measurement.
6. To develop a part of the tables of trigonometric functions and to use more complete ones.
7. To use the surveyor's transit if one is available.
8. To formulate rough checks on all solutions.
9. To construct the graph of the tangent, sine, and cosine of angles, from 0° to 90°.

OBJECTIVES IN TEACHING SIMULTANEOUS LINEAR EQUATIONS

To develop the following abilities:

1. To understand, by means of graphic representation, simultaneous, inconsistent, and equivalent equations in two unknowns.
2. To solve by addition and subtraction simultaneous equations in two unknowns, having integral or fractional (common or decimal) coefficients.
3. To obtain the equation of a straight line through two points whose coordinates are known.
4. To solve for the constants as well as for the unknowns in simultaneous literal equations.
5. To solve applied problems involving simultaneous equations in two unknowns.
6. To check all results.

OBJECTIVES IN TEACHING POWERS AND ROOTS

To develop the following abilities:

1. To understand the meaning of such necessary terms as base, power, root, principal root, rational number, imaginary number, and real number.
2. To solve certain important formulas involved, *e. g.*, in $A = \pi r^2$ we get $r = \sqrt{A/\pi}$.

3. To find roots both from a table and by computation.
4. To find roots without tables—by inspection and by short cuts, *e. g.*, by a graph.
5. To distinguish between arithmetic, algebraic, and geometric roots.
6. To find the square root of simple polynomials as an aid to an understanding of the corresponding arithmetic method.
7. To apply square root in simple problems relating to the Pythagorean Theorem.
8. To understand what is meant by finding roots to a certain number of significant figures.
9. To understand certain common exponential relations; e. g., $a^{\frac{1}{2}} = \sqrt{a}$, $a^0 = 1$, $a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$,
10. To do simple problems involving the four fundamental operations with surds.
11. To solve simple radical equations no more difficult than $\sqrt{2x+1} = 2x - 1$.

OBJECTIVES IN THE TEACHING OF QUADRATIC EQUATIONS

To develop the following abilities:

1. To construct, understand, and interpret the graph of the general case of a quadratic function $y = ax^2 + bx + c$ and thus to appreciate that the function $ax^2 + bx + c$ represents an entire family of values.
2. To understand that when $y = o$ the solution of the resulting quadratic equation amounts to finding the zeros of the function. The student should give both the graphic and the algebraic methods.
3. To understand that the form of the graph of a quadratic function is determined from certain known constants in the function.
4. To understand the significance of pure, incomplete, and complete quadratic equations.
5. To solve quadratic equations by factoring.

6. To solve quadratic equations by the method of completing the square.
7. To solve quadratic equations by the quadratic formula.
8. To understand simple graphic and algebraic methods of finding maxima and minima for quadratic functions.
9. To solve applied problems involving the use of the quadratic equation.
10. To check the solutions except in very involved cases.

The sentiment among the teachers who helped to make out the objectives is very strong for making the teaching of quadratic equations optional in the ninth grade. Many teachers favor the omission of the topic.

There is also a great deal of sentiment for making the teaching of logarithms and the slide rule optional in the ninth grade. Doubtless there are other topics in the preceding list which some teachers will wish to see omitted, and other topics not on the list which they may wish to see introduced. I hope that those who are interested in suggesting omissions or additions will do so through the columns of The Mathematics Teacher.

PART IV

A LIST OF DESIRABLE FORMULAS FOR USE IN NINTH GRADE ALGEBRA

SUBJECT	FORMULA
Area of a rectangle	$A = lw$
Area of a parallelogram	$A = bh$
Area of a triangle	$A = \frac{1}{2}bh$
Area of a trapezoid	$A = \frac{}{h}(b + b')/2$
Perimeter of a rectangle	$P = 2(l + w)$
Perimeter of a square	$P = 4s$
Perimeter of a triangle	$P = a + b + c$
Perimeter of an equilateral triangle	$P = 3s$
Circumference of a circle	$C = 2\pi r = \pi d$
Area of a circle	$A = \pi r^2 = \pi d^2/4$
Area of a regular polygon	$A = \frac{1}{2}ap$
Total area of a rectangular solid	$S = 2lw + 2lh + 2hw$
Total area of a cube	$S = 6e^2$
Lateral area of a circular cylinder	$L = 2\pi rh$
Total area of a cylinder	$S = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$
Lateral area of a circular cone	$L = \pi rl = Cl/2$
Total area of a circular cone	$S = \pi rl + \pi r^2$
Area of a sphere	$S = 4\pi r^2 = \pi d^2$
Volume of a rectangular solid	$V = lwh$

SUBJECT	FORMULA
Volume of a cube	$V = e^3$
Volume of a circular cylinder	$V = \pi r^2 h = Bh$
Volume of a circular cone	$V = 1/3 \pi r^2 h$
Volume of a sphere	$V = 4/3 \pi r^3$
Area of a circular ring or annulus	$A = \pi\ (R^2 - r^2)$
The lever	$w^1 d^1 - w_2 d_2$
Distance, rate, time	$d = rt$
Thermometer	$F = 9 \pi 5C + 32$
	$C = 5\ (F - 32/9$
Interest	$i = prt$
Percentage	$P = BR$
Amount	$A = p + prt = p\ (1 + rt)$
	$A = p\ (1 + rt)^n$
Falling bodies	$S = vt + \frac{1}{2}\ vt^2$
	$S = \frac{1}{2} gt^2 = 16t^2$
Selling Price	$S = M\text{-}D$
	$S = C + G$
Force	$F = Ma$
Pendulum	$t = \pi\sqrt{l \mid g}$
Cost of a number of articles	$C = np$
Pythagorean Relation	$c^2 = a^2 + b^2$
Division	$D = dq + r$
Gear of bicycle	$G = 28n/r$
Central angle	$C = 360/n$
Electric Current or Ohm's Law	$I = E/R$
Angle Sum	$a = b + c = 180^0$
Boyle's Law	$P = V_1 P_1 / V$ or $PV = K$
Charles Law	$V = V_1 \mathrm{T} /_1$
Speed of pulleys	$S = sd/D$
Diagonal of a cube	$d = \sqrt{a^2 + b^2 + c^2}$
Belting in a roll	$F = (d + d^1)\ \pi N/2$
Spread of gears	$S = SN/n$
Length of an arc	$l = \mathrm{N}\pi r^2 /_{360}$
	$0.017453NR$
Area of a sector	$A = N \pi d /_{360}$
Altitude of an equilateral triangle	$A = a\sqrt{3/2}$
Area of equilateral triangle	$h = a^2\sqrt{3/4}$
Diagonal of a square	$d = a\sqrt{2}$
Miles in feet	$M = 0.00019\mathrm{F}.$
Cube inscribed in a sphere	$E = 1.1547R$
Number of trees in an orchard	$N = nr$
Square of a sum	$(a + b)^2 = a^2 + 2ab + b^2$
Difference of two squares	$(a^2 - b^2 = (a - b)\ (a + b)$
Work formula for 2 men	$N = ab/a + b$
Arithmetic series	$l = a + (n - 1)d$
	$\mathrm{S} = \frac{1}{2}\ (a + l)$
Geometric series	$l = ar^n - \mathrm{a} \div r - 1$
	$S = (ar^n - a) \div (r - 1)$
	$S = (lr - a) \div (r - 1)$
Density	$D = m \div v$
Radio	$R = pl \div a$
	$W = \mathrm{EI}$
Power in Watts	$P = E \div r$

The following seven formulas for parcel post rates are simple, useful, and make a good group for comparative graphs. (They will need to be corrected when the rates change.)

First and second zones, up to 150 miles	$C = w + 4$
Third zone, 150-300 miles	$C = 2w + 4$
Fourth zone, 300-600 miles	$C = 4w + 3$
Fifth zone, 600-1,000 miles	$C = 6w + 2$
Sixth zone, 1,000-1,400 miles	$C = 8w + 1$
Seventh zone, 1,400-1,800 miles	$C = 10w + 1$
Eighth zone, over 1,800 miles	$C = 12w$

The following formulas represent some of the practical cases which one is likely to meet in various fields. The teacher will find them useful as supplementary material.

1. If a church is divided by a middle aisle into 2 sections, each having (p) pews, seating (n) persons in each pew, the total number of persons (T) that the church will seat is found by the formula: $T = 2pn$.
2. The velocity in feet per second (V) with which a wave is moving is equal to the length of the wave in feet (L) divided by the time (T) of one oscillation (rise and fall) of the wave in seconds. This is expressed by the formula: $V = L/T$.
3. In making a weak solution from a strong one a hospital nurse uses the formula $N = \frac{S}{W} - 1$ in which (N) means the number of gallons of water to be added for each gallon of the strong solution, (S) means the strength of the strong solution, and (W) means the strength of the weak solution.
4. The weight (W) of a square inch of metal if (g) is the weight of a rectangular piece of the metal a inches long and b inches wide is given by the formula: $W = g/ab$.
5. If w is the weight of a bag of marbles, b the weight of the bag alone, n the number of marbles in the bag, and m the weight of one marble, then the formula for the weight of the bag of marbles is: $W = b + mn$.
6. The formula for the area (A) of a stone wall to be whitewashed is $A = CD - cd$ where the length of the wall is C feet and the height is D feet, there being in the wall a window whose length is c feet and whose height is d feet.
7. If there are (S) subscribers in a telephone system, the number of connections (N) a central office can make is given by the formula: $N = S(S - 1)/2$.
8. In a business, the "turnover" is found by the formula $T = (C + C/n)/t$ in which (T) is the turnover, (C) is the capital invested, ($1/n$) is the fraction of a dollar that can be borrowed on each dollar of capital, and $t =$ the time in months required for the turnover.
9. The number of edges (E) of a geometric solid is given by the formula $E = F + V - 2$.
10. The pressure (P) of the wind in pounds per square foot is given by the formula: $P = 0.005V^2$ where (V) is the velocity of the wind in miles per hour.

11. The numbers of bushels of potatoes (B) in a rectangular bin is given by the formula: $B = lwh \div 1\frac{1}{2}$ where $1\frac{1}{2}$ represents the number of cubic feet occupied by 1 bu. of potatoes and l, w and b represent the dimensions of the bin in feet.

12. The average diameter of a tree D is given by the formula: $D = G/\pi$ where G represents the average girth.

13. The number of bricks needed in building a rectangular wall (B) is given by the formula $B = lwh/22$ where (l) is the length, (w) the width, and (h) is height of the wall in feet and 22 is the number of bricks in 1 cu. ft. of the wall.

14. The number of gallons (G) a rectangular cistern will hold is given bv the formula: $G = 7\frac{1}{2}\ lwh$, where l, w, and h are the dimensions of the cistern in feet and $7\frac{1}{2}$ represents the number of gal. in 1 cu. ft.

15. The number of tons of coal (T) in a rectangular bin is given by the formula: $T = lwh/36$ where l, w, and h are the dimensions of the bin in feet and 36 represents the number of cu. ft. in one ton.

16. The horse power of an automobile engine is given by the formula: $H = N D^2/2.5$ where D is the diameter of the cylinder in inches and N is the number of cylinders.

PART V

SELECTED BIBLIOGRAPHY ON OBJECTIVES IN THE TEACHING OF MATHEMATICS IN SECONDARY EDUCATION

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A discussion of the Report of the National Committee on Mathematical Requirements, 1923.

Brown University: *Suggestions for Students of Mathematics and Life Activities.* Bulletin of the Department of Mathematics, Sept., 1925.
Discusses the benefits that should be derived from the study of mathematics, gives suggestions as to methods of studying mathematics and gives an outline of mathematics and activities subsequent to college years.

Bobbitt, Franklin. *Curriculum Making in Los Angeles.* Chapter VII. Monograph 20, 1922. University of Chicago Press.
The chapter is devoted to a discussion of mathematics and a list of objectives is given.

Carmichael, R. D. *Mathematics and Life—The Vitalizing of Secondary Mathematics.* School Science and Mathematics, 15:105-115.
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Mayo, C. H. P. *The Position of Mathematics.. Educational* Review, 57:194-204.
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A statement of the contribution made by mathematics to the development of efficiency in education in the degree in which it develops the power of the individual. An appreciation of the aesthetic, and a demand for honesty and truth.

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The report is a discussion of what should be taught in mathematics, how much of it, to whom, how, and why. It contains, also, certain tentative suggestions for developing new and better courses according to the needs of the students.

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A brief outline of the report of the National Committee on Mathematics suggesting courses for the different grades.

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THE TEACHERS' COURSE IN ARITHMETIC IN THE NORMAL SCHOOL

A Plea for Professionalized Subject Matter

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What training is necessary to prepare high-school graduates to teach arithmetic and other subjects in the modern elementary school? This question has come to be a very important one in every normal school and particularly in those normal schools which still have a two-year course at the end of which the students accept positions to teach. This question is a difficult one to answer because teachers to-day need a much broader professional training than formerly. One attempt to solve this problem has been to increase the length of the normal-school course from two years to four years, but even in those schools where the four-year course exists comparatively few students remain to complete it, the large majority accepting positions after finishing the two-year or three-year curricula offered in those schools. Hence, how to prepare students in two or three years to teach effectively in elementary schools is still a vital question in practically all our normal schools.

Let us confine this discussion to the teaching of arithmetic and review for a moment the types of teachers' courses in this subject now being offered in many of the normal schools of this country.

If we view the matter in the large we shall find that the majority of courses now offered in these schools are of one of the following types:

(1) *Methods courses.* These courses are concerned with the details of teaching each topic in arithmetic, the class work often being supplemented by the observation of model lessons and the making of lesson plans.

(2) *Review courses.* In such courses the various topics of arithmetic are fully reviewed with more or less practice in problem solving. Since all students are fairly familiar with the elementary processes taught in the first four grades, this review largely centers upon the work of the upper grades, often stressing particularly percentage and its applications. Such a course

is finally of more service to those who teach in the intermediate or higher grades than it is to those interested in the primary grades.

(3) *Advanced courses in mathematics.* This work often consists of an advanced course in modern business arithmetic together with a formal study of the principles of arithmetic on a higher level. In some cases this work includes a review of topics in algebra and geometry and at times even treats of college algebra and trigonometry. Such a course has often been encouraged by a desire on the part of the administrative authorities to bring the normal-school work up to a collegiate rank. This type of course is of service to the prospective teacher of the upper grades or of junior high-school mathematics, but it does little to touch the problems of teaching arithmetic in the first five grades.

(4) *Courses in professionalized subject matter.* These courses attempt to combine the good features of (1), (2), and (3) above, aiming to look at arithmetic from a broader viewpoint, but at the same time so to treat each topic as to give definite help in teaching it in the elementary grades. Such a course if properly presented may be of far greater professional value to teachers than any of the courses mentioned above. Unfortunately this professional type of course is not frequently found in our normal schools to-day.

Combinations and variations of the above types of courses are also found, some schools combining types (1) and (2) while others offer (3) as a supplement to (1) or (2). We may summarize the matter by saying that in a recent survey covering 100 normal schools it was found that over 85 per cent of the teachers' courses in arithmetic given in this country are of types (1), (2), or (3), or variations thereof. Evidently courses of type (4), which represent professionalized subject matter, are only beginning to come into existence and there is a definite reason why this is so, as will be explained later in this article.

Considering this matter from another viewpoint we find that many of the teachers' courses in arithmetic of types (2) and (3) in particular, and sometimes of type (1), which are given in our normal schools to-day, are in general similar to those offered twenty-five years ago, although conditions in our elementary schools have changed very materially in that time. Let us review briefly what an elementary school teacher needed to know twenty-

five years ago compared with the demands made upon her to-day. In the first place, she did not have to study as many subjects as she does now because the curriculum of that time was not as varied as that of the present. Likewise she needed to know less of each subject than is expected to-day. In arithmetic, for example, her needs were then fairly well met by a thorough review of the subject with such instruction in methods as was available at that time. Such preparation, supplemented by practice teaching, served not only for arithmetic but for all other subjects. In contrast, the modern teacher of arithmetic must be far better equipped. She is expected to know the standard subject matter as thoroughly as formerly and in addition she must be able to teach such new topics as graphs, scale drawing, estimating answers, and how to check work in such a way as to make them interesting to pupils in the intermediate grades. In addition she must be thoroughly conversant with the actual applications of this material in modern business and industrial life. She must also be familiar with the modern testing movement in arithmetic and be able through standardized tests to discover the weaknesses of her pupils in the fundamental operations and to correct them through judiciously applied practice exercises. She must also know something of the results of certain educational experimentation which has made us conscious of the difficulties children have in learning certain topics and which has also shown us how to grade abstract problems in reference to their difficulty. In concrete problems she must take special care to see that the vocabulary is sufficiently simple and that the content of the problem is such that it appeals to children's interests. Further, she must know something of project teaching as applied specifically to arithmetic, taking particular care to keep her head level in this subject so that her teaching shall not be all projects at the expense of some of the fundamental skills of arithmetic.

Twenty-five years ago the textbook in arithmetic was the course of study. To-day the teacher must be equipped to follow and interpret a very elaborate course of study. The very comprehensive courses of study in arithmetic recently issued by the city schools of Denver, Detroit, and Baltimore cannot be intelligently followed without a training in arithmetic quite different from that which prevailed in our normal schools a quarter of a century ago. Each of these new courses of study contains over

140 pages and is limited wholly to instruction in arithmetic in the elementary school, two of them covering only the work of the first six grades!

Realizing the importance of this matter a group of instructors in arithmetic, representing eleven different normal schools along the Atlantic Coast, recently held a two-day conference to discuss possible improvements in the teachers' course in arithmetic. In each of these normal schools the problem is the same, namely, to offer in the two-year normal-school course such training in the teaching of arithmetic, geography, and other subjects, as will fit the student to accept a position in an elementary school at the end of that time. In each normal school the student body is made up of recent high-school graduates without experience in teaching. In our further discussion, therefore, we shall assume that the normal-school course under consideration covers only two years of instruction and that the students are recent high-school graduates. We shall also assume that we are preparing only for the teaching of arithmetic in the elementary grades and that we are not including special preparation for teaching junior high-school mathematics.

To assist in our further study of this problem let us first make a brief survey of the courses now being offered in the eleven normal schools represented in the conference.

In the majority of these schools the students must elect to specialize either in the primary or the intermediate grades and they are grouped accordingly throughout the two-year course. In general, the work in arithmetic follows this grouping, one course being given to the primary group and another course to the intermediate group. In a few schools, however, the course in arithmetic is the same for all students, that is, all are given preparation for teaching in all the grades of the elementary school. This question of grouping becomes, therefore, an important problem since the instructors reported that students trained for the primary grades not infrequently accept positions in the upper grades and vice versa. The topics and arrangement of the teachers' course in arithmetic differ considerably in the several schools, yet each course is an earnest attempt to meet conditions as they actually exist in their respective schools. Some schools have been including in their courses a considerable amount of what we have already called "professionalized subject matter"

in arithmetic which will be discussed more fully later in this article. In a number of schools a supplementary course in social-economic arithmetic is offered for cultural purposes and in some instances this is required of all students in addition to the methods course in arithmetic.

The problems reported by these instructors must be given the fullest consideration since each instructor has had considerable experience in this type of work and has also had thorough training for it. A complicating element of considerable importance, reported by each instructor, is the fact that many of the high-school graduates, on entering the normal school, are found deficient in their knowledge of arithmetic, which forces the normal-school course to include more or less review of the subject matter of arithmetic with such work in methods and other professional topics as the time may permit. The length of time assigned the course in arithmetic varies from a total of 60 to 120 hours. In several schools where 60 hours is the time allowed, the instructors feel that their present course is not satisfactory simply because of the limited time scheduled for it. The time allotted, however, is in many instances prescribed by the state authorities, thus giving the individual normal school no control in the matter.

There is considerable variation in the relation of the course in arithmetic to the work in the practice or demonstration school. In some instances the work in methods is supplemented by demonstrations in the several grades of the practice school, these demonstrations being made by the instructor of the methods course. In other instances the mathematics department has no contact whatever with the practice teaching or demonstration work.

Throughout the conference it was the opinion of the several instructors that it is important to include in the normal-school course as much of the professional treatment of the subject matter of arithmetic as is possible. By the phrase "professional treatment of subject matter" is meant such a presentation of each important topic of arithmetic as gives the teacher a broader vision concerning it and, at the same time, relates it directly to the teaching of that topic to pupils in the elementary school. A professional treatment of common fractions, for example, would include such information as would enable the teacher to present this subject most effectively in the intermediate grades.

Such a treatment would probably include a careful study of just what a fraction is, making clear that a fraction may be considered as representing parts of a single object, or parts of a group of objects, or as an expression of relation or comparison, or as a means of indicating division. The grades in which these several notions of a fraction are to be presented must be carefully discussed. There would then come, no doubt, a thorough review of the principles of fractions, including reduction, addition, subtraction, multiplication and division. This would lead to a consideration from the academic viewpoint of how difficult it is to explain satisfactorily our method of multiplying fractions or why we invert the divisor in the division of fractions, all of which forces us to define what we mean when we multiply by a fraction. After acquainting the normal-school student with what is a satisfactory mathematical treatment of this subject it becomes quite another question to discover what is a satisfactory pedagogical treatment of it and to learn how far the mathematical niceties must be abandoned in order to teach this subject effectively to young children.

There would then come a further discussion of the topic of reducing fractions to their lowest terms in which we discover that while a fraction like $\frac{255}{374}$ once needed to be reduced to its lowest terms by the tedious long-division or Euclidean method shown at the right, to-day such reduction is no longer necessary and consequently there is no need for teaching this method of finding the greatest common divisor of the numerator and denominator.[1] If such a fraction occurred practically to-day as it might in dividing 54,859 by 374, which gives a quotient of $146\frac{255}{374}$ we would continue the division, thus expressing the fraction in decimal form to the necessary number of decimal places. The result would then be 146.68 or 146.682 according to the degree of accuracy demanded. Such a result is far more usable than the fractional form given above.

```
255)374(1
    255
    ---
    119)255(2
        238
        ---
         17)119(7
            119
```

[1] By the method illustrated above 17 is found to be the greatest common divisor of 255 and 374 since in the process of continued division 17 was found to be contained in 119 without a remainder. Dividing both the numerator and denominator of 255/374 by 17 we get 15/22 as the result in its lowest terms. This method is still taught in many of our schools to-day which is a good illustration of how slowly changes in the curriculum are brought about.

Following the above discussions we treat briefly the history of common and decimal fractions, from which we find that in the larger transactions of life the common fraction is no longer as commonly used as the decimal fraction. This brings us to the question, What common fractions do we need to-day? This leads us to study several surveys, such as those by Wilson, Charters, Wise, and others, which were made to find out just what common fractions are used in the business life of to-day. From these we learn that common fractions with large denominators are no longer in general use and that the majority of practical needs are met with fractions whose denominators do not exceed 8 or 16. The question may then arise as to why we even have fractions with such denominators as 8 or 16, and then we point out the close relation between these fractions and our measuring systems. As soon as we stop dividing the pound into sixteen ounces and the inch into eighths and sixteenths, as would be the case if the metric system were used in this country, then we shall have little use for even such simple fractions as $^3/_8$ and $^5/_{16}$. This suggests that we examine the modern courses of study in arithmetic to see if they are up to date in respect to this topic, and we find with considerable satisfaction that they are since they are now prescribing that no common fractions of the more elaborate type shall be presented.

We are next led to a discussion of simple tests of divisibility because such tests now become important for the reduction of such small fractions as we need to-day since we have decided to omit the old Euclidean method of reducing fractions mentioned above. At this point comes an opportunity for a professional treatment of subject matter of unusual interest and quite different from any that has heretofore existed. If properly presented we are now able to discuss intelligently why we have tests of divisibility for such numbers as 2, 3, 4, 5, 6, 8, 9, and sometimes 11, and why we are never taught tests for 7 or 13. We then show the student how to make a test of divisibility for any other number, such as 17, and having made such a test he readily understands why it isn't practical to use it since it takes more time to apply it than to divide by 17 in the first place. This immediately gives us a foundation for a later discussion of the method of checking by casting out 9's, which is intimately related to the test of di-

visibility by 9, and then we fully understand, as we could in no other way, why this interesting check sometimes fails us.

With such orientation concerning the kind of common fractions to be taught to-day we now come to the consideration of the best methods of presenting them to elementary school pupils. This means an examination and study of the methods presented in the more prominent school texts, a review of certain experiments to improve the teaching of fractions, a study of typical errors which pupils make and the means of remedying them, the question of diagnostic tests and practice exercises in fractions, together with all the little niceties of explanation and motivation which go to make up the fine art of teaching. But the matter does not stop here. We then have the task of searching for problems, actually found in life, which apply fractions, and of selecting from a large group of such problems those which will be of special interest to children.

Thus we see that the "professional treatment" of fractions has been a combination of the usual study of the principles of fractions, a brief history of their development, and a scholarly proof of the tests of divisibility with their relations to checking by 9's and 11's, all of which has been supplemented by a discussion of modern usage in arithmetic, and a full consideration of those experiments which have contributed to the more effective teaching of this subject. A similar professional study can be given the other topics of arithmetic.

The materials for the professional treatment of any school subject are sometimes called "professionalized subject matter." We are just beginning to develop a satisfactory professionalized subject matter for arithmetic. Such a development would not have been possible twenty-five years ago because the contributions of educational experimentation and educational psychology to such a course were not then made; likewise certain materials on the history of arithmetic were not then in accessible form. The contributions of David Eugene Smith on the history and teaching of arithmetic, of the *Third Year Book* (1925) of the Department of Superintendence in giving the results of 48 experiments in arithmetic, and of Buswell and Judd in their recent book giving a *Summary of Educational Investigations Relating to Arithmetic,* have now made available source material which will be invaluable to all who are interested in creating a

new type of teachers' course in arithmetic. Considerable assistance will also be obtained by reading Chapter III of Randolph's dissertation entitled *The Professional Treatment of Subject Matter* in which the reader will find general principles for working out this idea for each individual subject of the curriculum.

In view of the previous discussion let us now consider what would be an ideal course for preparing a normal-school student to teach arithmetic. According to the combined judgment of the teachers of the conference the following seem to be desirable elements in such a course:

1. A professionalized treatment of the subject matter in arithmetic to include (*a*) a broad treatment and coördinating study of the basic principles of arithmetic, (*b*) the presentation of such advanced mathematical topics as will better equip the teacher for her work, (*c*) a consideration of the results of the most important studies and surveys relating to the kind of arithmetic needed for present-day life, an (*d*) a summary of the results of the more prominent experiments to improve the teaching of arithmetic.

Thus, each topic studied in the elementary school will be so treated as to answer such questions as the following: What are the important principles connected with this topic? To what advanced subjects in mathematics is this topic closely related? How does the world use this topic practically? What are the difficulties in presenting this material to children and what is a proper gradation of it? What are approved methods and devices for teaching this particular topic? Along with this work the normal-school student should be given practice in interpreting a modern course of study in arithmetic for the eight grades of the elementary school.

2. Demonstrations in the practice school in which typical lessons in arithmetic will be taught in the several grades. It is considered desirable to give such work, if feasible, soon after the class-room discussion of the topic in question.

3. The methods of measuring progress in arithmetic through standardized tests and of remedying the individual defects of pupils through practice exercises.

4. Some work in social-economic arithmetic covering those

phases of percentage, banking, thrift, investment, taxes, and insurance which more or less affect the lives of all average thrifty adults. Such a course is sometimes called modern business arithmetic though it might also be called "personal arithmetic." In those normal schools where students are grouped according to the primary or intermediate grades, such a course in business arithmetic will form a part of the professionalized subject matter presented to the intermediate group though it may be a supplementary course for the primary group.

5. A brief history of arithmetic for the purpose of showing why certain topics, once useful, are no longer taught and to fix the fundamental fact that the content of arithmetic, like that of the social sciences, is constantly subject to change.

6. Provision for bringing each normal-school student up to some standard skill in the fundamental operations. It was reported by several instructors that many students on entering the normal school do not have fifth-grade skill in the fundamental processes.

7. Provision for some practice in problem solving.

Items (6) and (7) in the above list are a complicating factor of considerable importance. It was felt, however, that (6) could be combined with (3), the normal-school students being shown how to improve their personal skill in the fundamental operations at the same time that they were being taught how to administer practice exercises to elementary school pupils. Similarly, the work in problem solving given in (7) could in large measure be provided in connection with the work in social-economic arithmetic mentioned in (4). While it is desirable to have the demonstrations in the practice school mentioned in (2) in close relation to the class-room work in methods, it is not absolutely necessary that such an arrangement be made. These demonstrations could come later if the normal-school schedule so prescribes.

As to the total time necessary for a course such as that outlined it was thought that from 100 to 120 class periods are needed.

For the coming year the instructors represented in the conference are planning to experiment with the course suggested above to see how far each of the elements mentioned may be included in the course they are now giving, each school attempting this so far as possible under its present time allotment.

There is an immense amount of work to be done before a satisfactory teachers' course in arithmetic can be devised, and the outline given above must be considered merely as a tentative suggestion to those who may be interested in such a problem.

For materials relating to this work the following books of reference are suggested. On the side of experimentation and tests this list may be supplemented by the very full bibliographies given in Buswell and Judd's *Summary* and in Chapter XIII of the *Report of the National Committee on Mathematical Requirements.*

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EDUCATIONAL EXPERIMENTATION AND STANDARDIZED TESTS

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HOTZ ALGEBRA SCALES IN THE PACIFIC NORTHWEST

By PROFESSOR WALTER CROSBY EELLS
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As a result of an address on "Standard Tests in Mathematics" given by the author at the annual meeting of the Inland Empire Council of Teachers of Mathematics at Spokane in April, 1924, a committee on "Standard Tests in Mathematics in the High Schools of the Pacific Northwest" was appointed. He was made chairman, the other members being Professor J. E. Buchanan, Cheney Normal School, representing the normal schools; and Miss Anna Whitney, Yakima High School, representing the secondary schools.

After careful consideration, this committee decided to concentrate its efforts for the year 1924-25 on a single set of standard tests for first year algebra. The series of shorter Hotz Scales (Series A) was selected as the best available for the purpose. In use the "B" scale was substituted for the "A" scale in "Problems" since the latter was not available.

A circular letter explaining the work of the committee and inviting co-operation was sent to about three hundred mathematics teachers in Oregon, Washington, Idaho, and Montana. Through special arrangements with the Bureau of Publications of Columbia University the tests were ordered directly from the committee. Tests were ordered by 89 schools, of which 45 were in Washington, 20 in Idaho, 15 in Montana, and 9 in Oregon. Reports were actually received by the committee from 67 of these schools. It is known that they were given in several more where reports were not sent. Approximately 25,000 copies of the tests were sent out.

The number of schools reporting on one or more of the tests was as follows:

Total Number Reporting	67
On Addition and Subtraction Test	64
On Equation and Formula Test	61
On Multiplication and Division Test	39
On Problem Test	38
On Graphs Test	20

These schools varied in size from enrollments of 2,300 students, to 5 students.

The number of different students reported as taking the tests were as follows:

Addition and Subtraction Test	4,168
Equation and Formula Test	4,195
Multiplication and Division Test	2,207
Problem Test	2,233
Graphs Test	801

An effort was made to have all schools give the Addition and the Equation Scales at the end of twelve weeks; the other three scales at the end of twenty-four weeks, for comparison with the standards given for those periods in the Hotz manual. Scoring was done by the different instructors, summaries only being sent to the committee.

A mimeographed compilation was sent to each co-operating school. Each school was known only by an arbitrary number, and the identity of it was not revealed to others. Each school knew its own number. Because the tests were given before the completion of the year, it was found to be fairer to arrange the schools into groups according to text books used. This would not have been so important if the tests had been given at the end of nine months, when the great difference in arrangement of topics would not have been so important.

Detailed Reports

The median score in each Scale, also the size of the high school and the number of students tested, is given in the table on the following page, in which the arrangement is according to text books used, and size of schools. Addition and Equation tests were given at the end of 12 weeks, other three tests at end of 24 weeks, unless indicated otherwise in parentheses, following the score.

School No.	Students in School	Students Tested	Addition Scale (A)	Equation Scale (A)	Multiplication Scale (A)	Problem Scale (B)	Graph Scale (A)
Perfect Scores			**12.0**	**12.0**	**12.0**	**14.0**	**8.0**
Hotz Standards		3 mo.	**5.0**	**4.9**	5.3	5.4	2.8 (4½m.)
		6 mo.	6.8	7.1	**6.3**	**6.5**	**(3.9)**
		9 mo.	7.9	7.8	7.9	7.5	5.6
Wells and Hart							
45	2300	330	5.9	6.1	8.5(25)	9.4(25)	
37	900	89	5.3	6.3	6.7	8.8	6.4(26)
23	550	192	5.2	5.5			
40	500	138	5.6	4.9	5.6(26)	6.1(26)	4.8(26)
80	350	101	5.6	5.8			
86	350	101	5.6(14)	6.3(14)			
53	300	77	6.4	6.0	8.3	9.0	
10	300	80		6.3			
82	300	102	5.5(13)	5.7(13)	8.7	7.4	
46	275	76	5.4(15)	5.9(15)			
57	275	71	5.6	6.3	6.8(25)	7.5(25)	
75	275	76	6.1	6.1	8.4(25)	6.7(25)	
66	250	66	6.2	7.2	8.3	8.8(25)	5.9
62	250	79	4.6(11)	4.9(11)			
5	225	55	6.2(13)	5.9(13)	6.8	6.7	
59	225	35			9.0(23)	5.7(23)	
84	175	42	5.4				
24	150	36	6.3	5.6	6.3(25)	5.4(25)	
39	125	41	6.4(11)	6.2(14)			
56	90	38	5.2	5.9	6.1	6.8	
7	90	19	3.4	5.8	7.8	7.5	5.6
14	75	29	5.6	5.8	8.6(25)	9.8(25)	
50	60	13	5.5	5.2	5.5(25)	6.9(25)	5.5(25)
4	60	16	4.8(13)	4.6(13)	6.4	6.0	4.9
74	40	9	5.5	5.2			
65	15	6	7.3				
Hawkes, Luby, and Touton.							
31	1500	264	6.2	6.7			
81	1300	232	5.[illegible]	5.6			
18	1100	246	6.3	5.6	7.9	6.3	6.1
36	650	119	6.0	5.4	7.7		
47	275	58	5.4	6.1			
77	200	57	5.3(13)	5.8(13)			
68	175	25	6.2	6.0	8.3	6.8	
88	125	45	(7.2) (26)	(5.4) (26)	7.8(26)	6.5(36)	
41	125	30	4.9	3.8	8.0	5.8	
25	95	25	5.1	6.1	8.7	9.8	5.8
3	65	13	5.4	5.8	8.5	6.3	5.7(27)
42	60	19	5.2	5.1			
16	50	19	5.8		7.2		
69	50	10	5.0(19)	5.7(19)			
76	40	15	6.1	4.9			
33	40	9	5.3	5.1	4.8	5.3	
34	30	5	3.5	4.3	5.5	4.5	5.0
58	5	2	5.5	6.0	6.0(25)	8.0(25)	1.0(25)

School No.	Students in School	Students Tested	Addition Scale (A)	Equation Scale (A)	Multiplication Scale (A)	Problem Scale (B)	Graph Scale (A)
Slaught and Lennes.							
51	300	77	4.7				
17	60	28	3.7	5.1			
20	60	16	6.3		4.8	6.8	
38	35	10	5.0	5.0	6.6(19)	4.7(19)	3.3(19)
Edgerton and Carpenter.							
71	850	174		6.0		6.7	
22	425	145	3.7	5.8(24)	4.9(27)	6.9(28)	
19	100	29	4.6	6.1	7.3	7.1	
Durrell and Arnold.							
78	260	44	4.3	4.6	6.1	6.6	
12	130	29	5.6	4.9	6.9(25)	7.9(25)	5.3(29)
6	75	23	5.4	6.1			
Sykes-Comstock.							
32	135	39	4.6(11)	4.9(11)	5.4	6.5	6.3
79	100	33	4.4	6.5	5.5(26)	8.1(27)	5.9(23)
Stone-Millis.							
2	100	32	4.6				
8	60	11	5.6(11)	4.9(11)			
Milne.							
13	50	13	4.5	5.5			
48	20	13	5.2	6.3	7.0(25)	6.2(25)	
Rushman and Dence.							
35	80	38	5.7	5.0	6.3(25)	5.4(25)	5.6(25)
Cajori and Odell.							
70	65	24	5.2(11)	4.5(11)			
Wentworth-Smith.							
27	90	28	4.8	4.6	7.0	7.9	6.5
Smith and Reeve.							
11	65	10	3.5	5.5			
Unknown.							
90	?	18	5.5	5.3			

General Summary

Five general summaries of this detailed report are given below, as follows:

a. General Summary.
b. Summary by Size of School.
c. Summary by Text Books Used.
d. Summary by States.
e. Summary by Sex.

A. GENERAL SUMMARY

(1) Comparison with Hotz Standards—Number of Schools.

Number of Schools	Addition	Equation	Mult.	Prob.	Graph
Exceeding Hotz Standard	44	45	25	23	15
Equaling Hotz Standard	2	6	2	2	0
Below the Hotz Standard	14	5	11	12	2

Of 31 schools giving the first four scales complete, 11 exceeded the Hotz standard in all of them.

(2) Comparison with Hotz Standards—Average of All Scores.

	Addition	Equation	Mult.	Prob.	Graph.
Hotz Standards	5.0	4.9	6.3	6.5	(3.7)
Average All Schools	5.4	5.6	7.0	7.0	5.2
Approximate Difference in Months	2-3	1	1¼	1½	2

B. SUMMARY BY SIZE OF SCHOOL

Definitions: Enrollment Over 300—*Large* High School.
Enrollment 100-300—*Medium* High School.
Enrollment Below 100—*Small* High School.

Average of Scores of Schools of Different Sizes

Class of School	Number of Schools	Addition	Equation	Mult.	Prob.	Graph.
Hotz Standards		**5.0**	**4.9**	**6.3**	**6.5**	**3.7**
Average All Schools		**5.4**	**5.6**	**7.0**	**7.0**	**5.2**
Large	16	5.6	5.8	7.3	7.6	5.8
Medium	21	5.5	5.7	7.2	6.7	5.9
Small	28	5.2	5.3	6.6	6.7	4.9

C. SUMMARY BY TEXT BOOKS USED. *Average Scores for Each Text.*

Text Book	No Schools	No Students Tested	Addition	Equation	Multiplication	Problems	Graph
Hotz Standards			**5.0**	**4.9**	**6.3**	**6.5**	**3.7**
Average All Schools			**5.4**	**5.6**	**7.0**	**7.0**	**5.2**
Wells and Hart	26	1917	5.7	5.8	7.4	7.4	5.5
Hawkes-Luby-Touton	18	1193	5.5	5.5	7.3	6.7	4.7
Slaught and Lennes	4	131	4.9	5.1	5.7	5.8	3.3
Edgerton-Carpenter	3	348	4.2	6.0	6.1	6.9	--
Durrell-Arnold	3	96	5.1	5.2	6.5	7.3	5.3
Sykes-Comstock	2	72	4.5	5.7	5.5	7.3	6.1
Stone-Millis	2	43	5.1	4.9	--	--	--
Milne	2	26	4.9	5.9	7.0	6.2	--

(For texts used in only one school each, see detailed summary.)

D. SUMMARY BY STATES. *Average of Scores for Schools of Each State.*

State	Number Schools	Addition	Equation	Mult.	Problem	Graph
Hotz Standards		**5.0**	**4.9**	**6.3**	**6.5**	**3.7**
Average All Schools		**5.4**	**5.6**	**7.0**	**7.0**	**5.2**
Washington	33	5.3	5.5	7.0	7.1	5.3
Idaho	16	5.3	5.5	6.4	6.7	4.9
Montana	10	6.0	5.8	8.3	7.7	5.7
Oregon	5	5.5	5.8	8.7	8.4	--

E. SUMMARY BY SEX. *(1) Average of Scores by Sexes.*

Sex	Addition	Equation	Mult.	Problem	Graph.
Boys	5.34	5.57	7.04	6.95	5.80
Girls	5.43	5.74	7.08	6.79	5.24

(2) *Comparative Rank in Schools.*

Number of Schools in which	Addition	Equation	Mult.	Problem	Graph
Boys' score is higher than girls' score	23	22	14	21	10
Boys' score is equal to girls' score	7	5	4	1	1
Boys' score is lower than girls' score	29	29	20	14	5

Special Groups

Reports of special groups of students from various schools show features that are of special interest in several cases.

a. Sections on Basis of Ability. Reports were received from three schools where sections were organized on the basis of ability into three groups, high, medium, low. Reports of these schools are given in the following table:

School No.	Section	No. Students	Addition	Equation	Mult.	Problem	Graph
36	High	24	6.2	5.7	8.8		
	Medium	79	5.8	5.3	7.1		
	Low	19	6.1	5.1	6.6		
57	High	19			7.4	9.5	
	Medium	22			6.5	7.1	
	Low	17			6.3	6.8	
71	High	23				8.2	
	Medium	64				7.2	
	Low	87				6.1	

b. "Leakage" of Information. In one school where there were two sections, taught by the same teacher, she reported that information had been passed on from the first section to the second, and therefore that the reports from the later section were unreliable. The extent of this "leakage" is suggested by the following comparative results:

Section	Mult.	Problem	Graph
First Section	6.3	5.4	5.6
Subsequent Section	8.8	8.0	5.1

Evidently graphic ability does not "leak" with the same facility as Multiplication or Problem solving!

c. Comparison of Regular Classes and "Repeaters".

School	Class	Number Students	Addition	Equation	Mult.	Prob.	Graph
25	Regulars	25	5.1	6.1	8.7	9.8	5.8
	Repeaters	10	6.3	6.4	10.0	9.7	6.5
39	Regulars	31	6.4	6.2			
	Repeaters	13	5.8	5.3			

d. *Reports on More Advanced Groups.*

School	Class	Number Students	Addition	Equation	Mult.	Prob.	Graph
82	Alg. 2 (30 wks.)	15	5.4	7.0	5.5	4.0(A)	
82	Alg. 3 (48 wks.)	21	6.9	10.0	8.5	6.8(A)	6.7
77	Alg. 2 (32 wks.)	20	7.1	6.5			
41	Alg. 3 (42 wks.)	11	9.3	8.6			
2	Completed last year (36 wks.)	7		9.8			

e. Comparison of Sections. Comparative scores of 16 sections with 8 different instructors, in a large high school of 2,300 students:

Addition. 7.1, 6.4, 6.3, 6.1, 6.0, 5.9, 5.9, 5.8, 5.8, 5.8, 5.8, 5.8, 5.8, 5.8, 5.6, 4.8.
Equation. 6.4, 6.3, 6.3, 6.2, 6.2, 6.2, 6.1, 6.1, 6.0, 6.0, 6.0, 5.9, 5.7, 5.6, 5.6, 5.3.
Multiplication. 9.1, 9.1, 9.1, 9.1, 9.0, 8.8, 8.7, 8.7, 8.2, 8.2, 8.2, 7.9, 7.8, 6.9.
Problems. 11.0, 10.7, 10.5, 10.3, 10.1, 9.9, 9.5, 9.4, 9.0, 9.0, 8.7, 8.5, 7.1, 7.0.

Addition.	Average	5.9	Average	Deviation	0.3
Equation.	"	6.0	"	"	0.2
Multiplication.	"	8.5	"	"	0.6
Problem.	"	9.4	"	"	0.9

Comments, Criticisms, and Suggestions

(A few extracts from letters from teachers who gave the tests.)

1. "The scale contains exercises which involve a study of fractions and radicals, which places it beyond the training of every pupil in a beginning class at the end of three months' work. I do not appreciate a scale that places half of the exercises under topics of which the pupils have never heard. It would hardly be fair to the pupils to test them on things which they have never studied."

2. "In our beginning classes the Hotz scales could not be used until toward the end of the second semester."

3. "I am heartily in sympathy with these tests. And I can truthfully say that they will be an aid to me in the teaching of algebra."

4. "My observation is that the end of 12 weeks is a very poor time to give this particular test (equation and formula)".

5. "The tests by no means cover the work required. They are too extensive."

6. "I don't seem to be able to figure out your idea of median score. You will no doubt pardon my stupidity."

7. "I believe the Hotz tests are very valuable, and hope you will carry on the work."

8. "We favor the continuation of the tests.—It would seem to me that a system of tests near the end of the year might be very valuable so that we could get a comparison of the results. As for geometry, let us try out the tests for a year at least."

9. "I hope that the mathematics committee will continue its work along similar lines next year. I suggest that the Hotz tests be given under similar conditions. Also I favor the introduction of a geometry scale. Would it be possible, in co-operation with other committees, to recommend a complete battery of tests for the more common high school subjects? Let them be handled in a similar way. I believe it would do much to get better teaching."

10. "I surely think it advisable to continue the activities of the committee. I shall repeat the use of the Hotz tests for Algebra next year and I am very anxious to give a geometry test for next year."

11. "We would like to see these tests continued next year, but would prefer to have a little more leeway in regard to time of giving them. We would be interested in a geometry test."

12. "I believe it is highly desirable for the committee to continue its activities another year. I would suggest that it might be very profitable to use the Hotz, the Rugg-Clark, and the Douglas tests under similar conditions. I believe it would be especially desirable to repeat the Hotz tests under similar conditions. I am confident they are an invaluable help in the improvement of teachers of algebra. I would be especially interested in a comprehensive test for plane geometry."

Summary and Conclusions

1. This committee received reports from sixty-seven schools in which about 4,000 students took one or more of the five Hotz standard tests in Algebra.

2. In each of the five tests about 75 per cent of the schools reporting, rank as high or higher than the Hotz national standards.

3. The average of all schools reporting indicate an achievement about one month in advance of the Hotz national standards

for three months' and six months' achievement. This advance is most marked in the Problem scale and the Graph scale where it is almost two months.

4. The large high schools have slightly higher scores than those of medium size, and noticeably higher ones than the smaller ones, but the total difference is not a marked one.

5. Considerable variety in scores is found for the different text books. Distinctly the best showing is made by the Wells and Hart text, which is used by the largest number of schools reporting, and by about one-half of the students reported. A comparison of text books, however, is not significant unless based upon results of tests at end of nine months, not of three and six months, due to different order of topics in the different books.

6. The best scores made (although the least reliable because representing fewer schools) are by the schools of Oregon and Montana. The poorest in all tests are made by the Idaho schools.

7. There is slight difference on the basis of sex. The girls make slightly better scores (average 0.1) on the three tests involving technical manipulation; the boys show a little more significant superiority (average 0.4) on the problem solving and graph tests.

8. For the average class of College freshmen, entering with one and one-half years of previous study of algebra, the instructor is not justified in assuming technical algebraic ability much in excess of that secured at the end of one year's study of algebra. For those with one year's preparation he is not justified in assuming much in excess of that secured at the end of one semester of high school algebra. (For justification of this conclusion, and other related ones, see special report, "What Amount of Algebra is Retained by College Freshmen," by the author in the MATHEMATICS TEACHER, Vol. XVIII, p. 219, April, 1925.)

The committee was continued for another year with an enlarged membership. It plans to continue work with the Hotz Scales, and also to promote the use of the new Sanford-Schorlnig Achievement Test in Plane Geometry.

There is a keen interest in the matter of standard tests in the states of the Pacific Northwest, and many other schools have indicated their intention of co-operating in the plan during 1925-1926.

MATHEMATICS IN METHODS

By PROFESSOR RALPH BEATLEY
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There is almost no one of average intelligence who cannot understand mathematics; yet many fail to. Why?

First, because from their teachers, their homes, their friends, and from their own inner consciousness they have built up incorrect notions and habits of thought about mathematics and mathematical processes, and every day's exercise but serves to strengthen these wrong habits.

Second, because their teachers do not succeed in bringing to light all these difficulties and frequently do not recognize them as incorrect when they are brought to light.

Third, because all too frequently these pupils come from homes where their mathematical shortcomings are condoned by parents who themselves "never could do mathematics."

If the parents who "never could do mathematics" would get the idea that their children in all probability can do mathematics, and that mathematics today is better taught than heretofore; and if they would encourage their children really to think through their mathematical difficulties and to press their teachers continually for explanations, and for more explanations, and for still more explanations,—that is, if they would only make the teachers teach; then one large source of our troubles would be removed.

The teachers too should encourage the children to state their troubles and their doubts, and should take advantage of the fact that the pupils' questions afford them their greatest opportunity for real teaching. But no teacher can be expected to answer satisfactorily every question which arises, whether at once or after several days of study. It does no harm to leave a doubt unanswered for the present; it does do untold harm to quote an explanation which the teacher herself cannot fathom. What better attitude can we hope to instill in our pupils than that they question everything which is not clear to them? Do we for a moment wish them to believe a thing simply because their teacher or the book says so? Better by far to leave a child unanswered

than to answer him incorrectly or insincerely. Why not rather encourage him to keep a list of the questions for which he has as yet no satisfactory answer? This is science; and it is teaching.

So much for the child's attitude toward subject and teacher. The rest of the story is concerned with one big problem: how best to lead children to form correct mathematical habits.

This problem has two aspects; one psychological, the other mathematical. The philosophy of mathematics and the psychology of learning both share in determining the nature of the several mathematical abilities, their relative difficulty, their proper arrangement in a course of study, and the most economical way of acquiring them. Thorndike and others have made important studies on the psychological side, a comparatively new and difficult field. I shall limit myself here to the mathematical side, but in what I shall say there is nothing new.

Many wrong habits developed by pupils have their origin in wrong mathematical ideas. For example, the perfectly correct notion that in mathematics we try to generalize as much as possible and state a proposition in terms so accurate and broad that the very statement of it shall embody all the special cases and apparent exceptions to it, has given rise—quite naturally—to the incorrect notion that in mathematics there are no exceptions whatsoever, and no special cases. Or again, the notion that we like to prove a body of propositions with the help of as few definitions and preliminary assumptions as possible, has carried over into the notion that in mathematics we can prove everything. The important and fundamental role of the definitions and preliminary assumptions has been further weakened by the frequent assertion that they are self-evident and obviously true; they are neither. The pupil should get the idea, rather, that—try as we will—we cannot get rid of all the exceptions; and that some few things must simply be taken for granted, just for the sake of argument. We must agree on a few fundamental definitions and assumptions before we can start to prove anything at all. It is not because these statements are so obvious or so easy to prove that we do not bother to prove them; the reason why we do not prove them is simply that we cannot. No more can we attempt to give a meaning to every word in the English language without eventually ending up in the situation, "that" means "which" and "which" means "that."

"What is seven divided by zero?" Up go the hands: "Zero"; "one"; "seven"; "infinity"! It seems hard to believe that the question does not admit of an answer; but it doesn't. We may not divide by zero. For if we could, then the quotient times zero would have to equal the dividend; and in multiplication we learned that q times 0 is 0. These two rules which prevent us from dividing by zero are in reality pure definitions. They were set up quite arbitrarily as the rules for our game of arithmetic; and so long as we play this particular game we must abide by them. In this game the rules permit us to add, to subtract, or to multiply any two integers, including zero. In division we may use any number, including zero, for the dividend; but for the divisor any number—*except zero.*

We can change some of the rules and make a new game, a new arithmetic with slightly different definitions. In this way we can arrange to make division by zero permissible. But we should have to admit some other exceptions, such as q times $0 \neq 0$, for instance.

The idea of other arithmetics with different definitions need not startle us. For even without changing the definitions, but by simply changing the base 10 to 5, the simplest and most familiar operations of our everyday arithmetic take on a bizarre, though equally valid, form. Thus $4 + 4 = 8$ and $4 \times 7 = 28$ become respectively $4 + 4 = 13$ and $4 \times 12 = 103$ when the base of the number system is 5.

"But I thought zero could also be regarded as a very, very small number, and that then it was correct to say 'seven divided by zero equals infinity.'" In our arithmetic we already have one helpful definition of zero, or 0, namely, that it is the symbol which stands for *a* minus *a*. Can we give to this same symbol another meaning or interpretation which is incompatible with its definition? Can it mean "exactly 2 minus 2" and at the same time "almost 2 minus 2, but not quite," "absolutely not any at all," and at the same time "a very wee little bit"? Even to admit this absurdity for a moment does us no good, for infinity is not a number; it is not a symbol for any one thing in particular. The most definite thing about infinity is its great indefiniteness. In our arithmetic, therefore, division by zero is impossible. It is an exception. Let us be honest and admit it.

For each value of x other than 0, $\frac{7}{x}$ has a perfectly definite value which we can compute. As x approaches 0 as its limit, $\frac{7}{x}$ increases indefinitely; it has no limit. This situation is frequently briefly described in print (though quite inadequately) by the following pseudo symbolism, $\lim_{x \to 0}(\frac{7}{x}) = \infty$, which carries with it two false implications: (1) that $\frac{7}{x}$ has a limit as x approaches 0; and (2) that ∞ is a symbol standing for something definite which could conceivably be the limit of something else. The best we can say then for the degenerate $\frac{7}{0} = \infty$ is that, as commonly read, it has no meaning. There seems to be no reason for encouraging it to live. So long as it remains with us, however, it may conceivably be of some slight help to us, provided we remember that it is essentially a living lie, that its left side is a fraud, and that its right side is a fake; but that in its feeble little way it is trying to bear witness to the truth that, as x approaches 0, $\frac{7}{x}$ increases without limit, and that, when $x = 0$, the symbol $\frac{7}{0}$ has no meaning whatsoever.

It is easy to see that the expression "$\tan 90° = \infty$" suffers under similar limitations.

Other troubles of a somewhat different nature arise from a failure to grasp the full significance of the postulate that space and time are infinitely divisible. This means that if we take a line one foot long, and mark its midpoint, and the midpoint of each equal segment, and the midpoint of the resulting four equal segments, and so on, the process may be continued indefinitely. By such a process therefore we could never arrive at a segment of zero length, nor at that intuitive something which we call a point. Each little segment is an infinitesimal, approaching, but never reaching, 0. Conversely we may regard a line of finite length as composed of an infinite number of such infinitesimal segments. We may not, however, say that an infinite number of zeros add up to something, nor that a line is simply the sum of an infinite number of points. Neither is a circle to be regarded as a regular polygon of infinitely many sides, nor a straight line as a circle of infinite radius.

According to Zeno in his famous paradox there was no logical explanation for the physical fact that Achilles could ever catch the tortoise, even though he ran uniformly ten times as fast as the tortoise, and with only 100 yards between them at the start.

For Zeno's contention, when resolved into a numerical case with modern notation, amounted to this. The tortoise at a given instant had proceeded 11.111 yards; Achilles at the same instant had covered 111.11 yards; and 111.11............could not equal 100 + 11.111........, where the decimal ascribed to the tortoise had always the same number of digits (regardless of the decimal point) as that ascribed to Achilles. Refusing, as he did, to admit that space and time were infinitely divisible, he virtually refused to admit that $100\frac{1}{9}$ expressed as a non-ending repeating decimal could equal $100 + \frac{1}{9}$ expressed as a non-ending repeating decimal.

* * * * *

Similar considerations affect our teaching of "(—5) times (—3) equals + 15," which is a pure definition and obstinately resists all efforts to prove it. Still further extensions of our number system to include irrationals and other "imaginaries" involve other pitfalls for the unwary teacher or pupil.

* * * * *

Many, many times a little deeper insight into the full mathematical significance of a situation or topic would change that topic from hard to easy. For, mathematics thoroughly understood and clearly presented is not hard. If here and there we have to skip a beat in the reasoning, that should not make the subject harder; unless, of course, we fail to let our pupils into the secret. If the collapse of the logical sequence at a given point makes it necessary to introduce a new definition or a new assumption before we can proceed, that fact should have proper emphasis. And if the pupil, for his part, will always keep clearly in mind, and in two separate categories, those mathematical statements which are fundamental definitions and assumptions in arithmetic, algebra, and geometry, and those mathematical truths which can be shown to follow logically from these definitions and assumptions, then he will see what *is* mathematics, and he will like it, because it is clear to him.

NEWS NOTES

The Editor desires a limited number of copies of the January, February and March, 1924, numbers of the Mathematics Teacher. If any reader is willing to supply any one of these copies, the Editor will advance the expiration date of his subscription two months for each copy that is sent in.

The Association of Teachers of Mathematics in New England held its annual spring meeting at Boston University Saturday, May 2, 1925. The program included: (1) The Teaching of Mathematics in an English Secondary School, by Miss Margaret Brown, Senior Mathematical Mistress, Girls County School, Bishop Aukland, England (exchange teacher, Bradford Academy); (2) Meeting the Attacks on Algebra, by Miss Mary J. Quigley, Teachers College, Boston; (3) Mathematics at a College of Business Administration, by Charles F. Stratton, Head of Mathematics Department, Boston University, College of Business Administration; and (4) The Meaning and Educational Value of the History of Mathematics, by George Sarton, Sc. D., Founder and Editor of "Isis" and Lecturer on the History of Science, Harvard University.

The regular spring meeting of the Philadelphia Section of the Association of Teachers of Mathematics of the Middle States and Maryland was held Saturday, March 21, 1925, at 9:30 in Central High School. The president, Mr. MacCormack presided.

The minutes of the October meeting were read and approved.

The treasurer's report of the dinner held March 12th, 1925, and the financial report of the year were read and accepted.

The report of the nominating committe was given by the chairman, Dr. Shoemaker, and accepted. The secretary was instructed to cast an unanimous ballot for the following officers: President—Mr. Samuel K. Brecht, Central High School; Vice President—Dr. O. E. Glenn, University of Pennsylvania; Secretary and Treasurer—Miss Edith E. Morin, West Philadelphia High School for Girls; Member of Executive Committee—Mr. Donald E. MacCormack, William Penn Charter School.

Dr. John R. Clark, Editor of the Mathematics Teacher, was the first speaker of the morning. He told of the purpose of the National Council of Teachers of Mathemaitcs and of the advantages to be gained if the Philadelphia Section became affiliated with it.

Professor C. B. Upton of Columbia University was the second speaker. His subject was "High School Mathematics as An Aid to the Appreciation of Life About Us."

A short discussion followed Dr. Upton's address after which it was moved and seconded that the question of the affiliation of the Philadelphia section with the National Council be left to the Executive Committee with power to act.

The Alabama Association of Teachers of Mathematics met in Mobile, Alabama, April 3, 1925. In the absence of the president, Mr. W. H. Fagerstrom, of Mobile, presided, and Miss Kirk, of Montgomery, acted as secretary pro tem. The report of the president was read, and also a report of the secretary and treasurer. Mr. Bandman's paper was read by Miss Sclater, of Mobile. The department then held a round table discussion for freshman mathematics. Then adjournment.

Professor H. S. Everett, of Bucknell University, reported at a recent meeting of the Pennsylvania Academy of Science that the departments of mathematics in the colleges of Pennsylvania show a marked interest in courses in finance and statistics. Twenty-three of the forty-one colleges that replied to his questionnaire are now giving or plan to give in the immediate future such courses. In concluding the report, Professor Everett states, "As we realize the fact that every refinement and ultimate crystallization of scientific developments depends for the final expression of its order and form on mathematical terminology and logic, as we realize that such expression is the final goal of each science, as we realize that, as Professor Swartzel of Pittsburgh puts it, the physicist has stolen the static atom of the chemist and made of it the dynamic atom of the physicist" because he had the mathematics with which to do so, as we realize the rapidly growing demands of political, economic, social, and biologic sciences on mathematics, evidenced for example in the fact that there is being offered this summer a course in mathematics entitled, The

Mathematical Theory of Economics, for which acquaintance with first order differential equations is a prerequisite, as we realize that we are no longer obliged to turn to foreign countries for our actuaries, for our sole source of statistical and economic theory, as we realize that the provision of elementary mathematics courses pointing the way to all comers that leads to better science, scientific method, and scientists is rapidly becoming more ample. I begin to entertain hopes that the intrusion on the program of an Academy of Science of paper essentially a curricular study and prospect will not need an explanation nor an apology.

The question of class-size and the teaching load in high school has become so pressing that the National Association of Secondary Principals have appointed a commission to study the problem. The members of the commission are as follows: C. P. Briggs, Principal High School, Lakewood, Ohio; H. V. Church, Principal J. Sterling Morton, High School, Cicero, Illinois; Earl Hudelson, University of Minnesota; F. S. Breed, University of Chicago; C. A. Fisher, Principal of High School, Kalamazoo, Michigan; M. H. Stuart, Principal Technical High School, Indianapolis, Indiana; and P. R. Stevenson, Ohio State University.

The first step in an investigation is to collect some empirical data concerning different possibilities for making efficient use of large classes. One plan is to give the teachers a large teaching load (40 to 55 pupils per class and five to six classes.) A clerk will be hired to devise tests, grade papers, and report pupil progress to teachers for remedial instruction. Another plan is to arrange for several sections in a given subject to meet together for one or more hours per week. For example, Civics or American History pupils might be divided into two or more sections and at specified times, these could be brought together in a large study hall or auditorium. One teacher should then demonstrate or present facts to the entire group. Such group meetings should be arranged whenever advisable and not necessarily at definite intervals.

Recent investigations have shown little or no advantage for small high-school classes. It is quite possible that teachers do not have a suitable technique for teaching either large or small classes. The commission will, therefore, investigate different

means of handling the two types of classes and endeavor to set up techniques which have proved themselves to be of advantage for large or small classes.

After the preliminary steps of the investigation, several controlled experiments will be conducted. Some of the problems which will be investigated in a scientific way under controlled conditions are as follows:

(1) The effect of giving the teachers a large teaching load and hiring clerical assistance.

(2) The advisability of instructing large groups for part of the time.

(3) The relative efficiency of large and small classes for bright, average and dull pupils when pupils are classified according to ability.

(4) The relative efficiency of large and small classes which are composed of pupils not classified according to ability.

(P. R. S.)

Professors David Eugene Smith and William David Reeve, of Teachers College, have published, with Ginn & Company, a second course in Algebra, entitled "Essentials of Algebra Part II."

Edward I. Edgerton, of Dickinson High School, New Jersey, and Perry A. Carpenter, of West High School, Rochester, have published with Allyn & Bacon a second course in Algebra entitled "Intermediate Algebra."

Mr. Harry C. Barber, of the English High School, Boston, is the author of the new text, "Everyday Algebra" for the ninth school year, recently published by Houghton Mifilin Company.

Miss Mary A. Ward, of San Francisco State Teacher's College, is the author of "Pupil's Self-Instruction Arithmetic," comprising three volumes: Formal and Problem Percentage, Application of Percentage, and Mensuration, published by Rand-McNally Co.

Doctor Winona Perry, Associate Professor of Educational Psychology in the University of Nebraska, has published her "A

Study in the Psychology in Geometry," with the Bureau of Publications, Teachers College, Columbia University.

Doctor John R. Clark and Doctor Arthur S. Otis, in co-operation with some 30 schools, are conducting an investigation in the teaching of plane geometry, the purpose of which is to determine the efficiency of certain methods of teaching the basal propositions of geometry. The experimental materials which are being used in the investigation are published by the Lincoln School.

The Mathematical Association of America held its ninth summer meeting at Ithaca, New York, September 8-9, 1925. The program included: Elementary Geometry and its Foundation, by Mr. H. E. Webb, Central High School, Newark, N. J.; The Mathematical Basis of Art, by Professor G. D. Birkhoff, Harvard University; Certain Applications of Differential and Integral Calculus in Actuarial Science, by Professor H. L. Rietz, University of Iowa; The College Entrance Examination Board From Behind the Scenes, by Professor Virgil Snyder, Cornell University; Outlines of Fields of Research; Projective Geometry, by Professor L. W. Dowling, University of Wisconsin; and The Mathematical Problems which Arise in a Research Laboratory, by Dr. T. C. Fry, Western Electric Company, New York.

NEW BOOKS

Plane Geometry (revised edition) Palmer-Taylor-Farnum, Scott, Foresman Company, 1925.

This revision of the original Palmer-Taylor Plane Geometry retains the features of the earlier book, including an experimental approach to demonstrative work, many practical exercises and considerable use of algebra, and differentiations in typography. The teacher is enabled to determine at a glance the propositions that are considered fundamental by the National Committee and those that are required by the college entrance board.

The Elements of Mechanics. By F. S. Carey and J. Proudman. Longmans, Green and Co., London, 1925. Pp. x + 314.

This is the most recent volume in the Longmans' Modern Mathematical Series, a collection of books that has done much to set forth the modern purposes and methods in the teaching of pure and applied mathematics. It is written by Professor-Emeritus Carey and Professor Proudman, both of the University of Liverpool. Professor Carey has long been known for his contributions to the study of the calculus and mechanics and for the high standard set by him when directing the work in these subjects in his university, and his collaborator has been prominent in the same field. The book represents, therefore, an authorship that assures a scholarly treatment of the subject.

The work is intended for use in the first year of the university, but it represents a standard that is hardly reached as early as this in our American colleges. It does not require a preliminary study of the calculus, but it presupposes what we do not generally have in this country, namely, a good course in mechanics in the secondary school.

While the chapter headings naturally represent the classical topics, the presentation of these topics represents a considerable departure from classical methods. The theory in general follows a statement of the problem to which it applies, proceeding from simple facts well known to the reader to the mathematical ex-

planation of these facts. This is seen in the case of speed and its measurement, of velocity-acceleration, of gravity, and so on through the list. Much attention is also paid to simple laboratory practice, a feature which British teachers have been able to carry out with much more attention to mathematics than is usually the case with us.

In the matter of arrangement in presentation the authors have adopted a plan that is worthy of attention. In the chapters there is printed a general treatment of the topic; then the contents refers to "Worked Examples" and "Examples" in the second half of the book. In this way they combine a text of theory and a book of exercises. Whether this is as good as our custom of combining the examples with the text is probably answerable only by considering the habits of the users.

Perhaps the most striking feature of the treatment is to be found in the use of vectors and in the well-arranged set of exercises on the subject.

The English writers have for a long time excelled in their exercises in applied mathematics. This fact strikes an American reader as he looks over the work under review. There are more than a hundred pages devoted to applications. Since the type is small, they represent the equivalent of 150 to 200 pages of our ordinary textbooks. Such a supply, including a large amount of new material, will prove very helpful to American students.

The entire work is worthy of careful reading and it seems not improbable that it will find a place as a textbook in some of our colleges.

DAVID EUGENE SMITH.

Intermediate Algebra, by Edward I. Edgerton, B. S. Dickinson High School, Jersey City, New Jersey, and Perry A. Carpenter, Ph. B. West High School, Rochester, New York. Pp. IV + 386. Allyn and Bacon, New York.

The authors of this text are evidently very well acquainted with the youth of immature minds, for they have provided the teachers with ample assistance. The first chapter of this book is devoted entirely to a review of fundamental operations. The subject matter is so simple, the principles so plainly stated and as abundantly illustrated, that any pupil may teach himself such elementary algebra as he has forgotten. There are abundant drill

problems if the teacher desires to review Course One in algebra in class. This feature is carried throughout the book, every new subject beginning with a review and leading logically into the more difficult phases. The last chapter is a complete summary of the contents of the book.

The problems are many and varied. There are examples taken from geometry and physics, from practical mechanics, and from everyday life, such as the automobile. There are also many of the historical problems,—mixture of elements, work and day and puzzles which are justified on account of their interest to the pupils. Some examples are quoted from Teachers' Entrance Examinations, Regents' Examinations, and College Entrance Requirements.

The function concept is developed and emphasized; a great variety of simple and difficult formulas is given together with their manipulation; the graph is treated in an interesting way both theoretically and practically and logarithms receive an entire chapter.

This text prepares the student for the more difficult and abstract work of mathematics in college.

Sophia R. Refior
Scott High School, Toledo, Ohio

Plane and Spherical Trigonometry, by C. I. Palmer, Associate Professor of Mathematics, Armour Institute of Technology and C. W. Seigh, Associate Professor of Mechanics, Armour Institute of Technology. Publishers: McGraw-Hill Book Co., New York. Pp. XIV + 136.

For the trigonometry teacher who has classes composed of pupils of varying ability, it is certainly advantageous to have a text which is adaptable. From this text, the teacher may have a long, vigorous course, if his class warrants such a method. However, if the time and ability do not make it feasible to cover the entire book, at least five chapters may be omitted without destroying the sequence.

This feature of adaptability is further marked by long lists of graded exercises as well as many oral problems. The instructor who likes to drill on principle will be delighted with this oral work. The simple problems are easy enough to please the weak pupils while the more difficult can challenge the strongest student.

The problems are of a very interesting and practical type, prominence being given to those on surveying and navigation.

The book abounds in many clear and explicit explanations, giving detailed directions for solving problems, including many illustrations.

The graphical representation of the trigonmetric functions, the drawings accompanying the practical problems as well as those cn complex numbers are especially good. The explanations of oblique triangles, complex numbers and of logarithms are very clear.

This text gives us the best material that has thus far been found. It does not give us much that is new or startling. It does not give us an easy practical development of intuitional trigonometry, such as might be used in junior high schools.

Sophia R. Refior
Scott High School, Toledo, Ohio

The History of Mathematics in Europe, by J. W. N. Sullivan. The Oxford Press, American Branch, New York. Pp. 110.

This volume of the *World's Manuals* is a neat little book, giving a simple but scholarly introduction into the history of mathematics. It is written as one consecutive human narrative and is well supplied with portraits, thus holding the reader's interest from beginning to end. Its scope is moderate and its style simple and hence it has an appeal to the general intelligent reader.

It is also accurate and scholarly and hence has a service for the mathematical specialist. To him, this book furnishes an outline of the history of mathematics by offering to him in concise form, accounts of the beacon lights of history. The book is divided into ten great divisions, each heading being written at the top of each page; each subtopic is indented and the names of every mathematician and his works put in italics so it can be used as a reference book with great facility.

The author has given numerous specimens of actual solutions and of notation and some facsimile pages, thus enabling the real student to see for himself what the early mathematicians accomplished. The biographies contain a brief account of the lives of the men with a more elaborate account of their works, depending on their relative importance, thus Descartes receives four pages while Newton is described on sixteen pages. The book would be

valuable to advanced high school students and to mathematical clubs and hence is strongly recommended for high school libraries.

SOPHIA R. REFIOR
Scott High School, Toledo, Ohio

Plane Geometry, by Royal A. Avery, North High School, Syracuse, New York. Allyn and Bacon. Pp XXV + 319.

This modern geometry considers the pupils interests and abilities. It is printed in attractive plain type with the figures standing out clearly, thus eliminating all eye strain. Interspersed among the pages are beautiful reproductions of fine architecture, illustrating the practical application of geometry.

In place of long lists of definitions, questions are asked, thus leading the pupil into the idea. Then the proofs grow out of the ideas and axioms, thus holding the pupil's interest and leading him into the rigorous part of geometry before his curiosity wanes.

Before the more difficult theorems are presented, simple problems illustrating parts of those theorems are given, thus preparing the pupil for the proposition. At the beginning of each theorem, is a plan of attack, so the pupil need not flounder around aimlessly, but make a direct, logical investigation.

The book, in all its phases, shows a keen sympathy for the pupil in selecting such problems as appeal to him instinctively. It also shows an understanding of his mind in the choice of material, the order of presentation and the amount of difficulty given him in every step.

This text, from one survey, seems one of the most teachable of the newer books, whether put into the hands of an experienced instructor or of a novice.

SOPHIA R. REFIOR
Scott High School, Toledo, Ohio

Essentials of Algebra, Book II, by Professor David Eugene Smith and William David Reeve. Ginn and Company. pp VI + 274.

This text is designed for a third semester in algebra in either the tenth or eleventh grade. It embodies the latest developments in education not only in the selection and arrangement of material, but also in its presentation.

Graphs are shown in their relation to the equation and the function of x. Their classification by types is shown by illustration. The graph is also used to introduce maxima and minima, thus preparing the student for the calculus.

Directed numbers, determinants in connection with linear equations, and the trigonometry of the right triangle receive ample consideration. In the treatment of logarithms, the table gives the entire number, characteristic as well as mantissa, thus insuring greater accuracy and facility.

The applied problems are interesting and practical. Discount, food values in a daily diet, parcel post and the automobile are among the subjects used. The formulas contain applications within the pupils' own experience, those used in physics or in business.

The numerous tests include the "True and False," "Recognition," and "Completion" types, and a safe general test with no time limits.

This text has many reviews. Each subject is preceded by a preliminary review of elementary algebra and is followed by a comprehensive review of its present treatment. There is also a general review at the end of the book.

SOPHIA R. REFIOR,
Scott High School, Toledo, Ohio.

Introduction of Algebra into American Schools in the Eighteenth Century, by Lao Genevra Simons, Bulletin 1924, No. 18, Government Printing Office, Washington. Price, 15c.

Doctor Simons' essay is of interest to students of the history of education as well as to those who are concerned primarily with mathematics, for the eighteenth century covers the interval from the first mention of the teaching of algebra in an American college (Yale, 1718), to the time when it had become a recognized

part of the curriculum. It should be noted that in the year following the reopening of the college after the Revolutionary War, Columbia required the study of algebra to quadratics for the freshman. It was not until 1820 that any American college required the subject for admission.

According to Miss Simons, the introduction of algebra was due to imitation of the English universities and to the influence of men trained in them. It was taught as a preliminary to work with fluxions (calculus) or for the mental enjoyment that it gave, but we cannot claim that it was taught for its utilitarian value. Although the point is not stressed in this essay, it would be well to remember that considerable material which we would normally put into an algebra was formerly classed as arithmetic, for many of the problems that we would solve by algebraic equations were then frequently met by the Rule of Three and other arithmetic methods. The fact that utilitarian values did not operate in the introduction of algebra into our schools and colleges two hundred years ago, should not be interpreted to mean that the subject has no practical value to students today. The two conceptions of algebra are fundamentally different. The older attitude is expressed in one of a group of statements or "theses" proposed for discussion by the graduating class of Yale in 1718,—the first mention of algebra in connection with one of our colleges.

"Algebra is the doctrine in which by comparing known quantities with unknown, *difficult* questions of arithmetic and geometry are easily resolved."

The statement was proposed in Latin, and the italics do not appear in the original.

Our information as to the scope of the work in algebra is gained from various sources. Among these are the "Commencement Theses" mentioned above. The first of these to concern algebraic subject matter at Yale were those of 1718; at Harvard, those of 1721. Another source is the manuscript notebook. Although these are rarer than the ones in arithmetic, copies exist of notebooks prepared by students of Harvard, Princeton, and the University of Pennsylvania. Those written under the same instructor though at different dates show great similarity. These sometimes contain references to the immediate source of the material, but they give little hint as to the conduct of the course or the conditions under which the notebooks were prepared. A third

source is a collection of sets of problems whose solutions were written in exhibition form by Harvard students in the last two decades of the century.

No volumes wholly devoted to algebra were printed in this country during this period, but certain arithmetics as those of Peter Venema (1730) and Nicolas Pike (1788) contain short sections on this subject. These supplements and the newspaper advertisements of tutors prepared to give instruction in it, indicate the interest taken in the subject of algebra. The content of the work may be judged from the citations given from these notebooks in algebra.

It would be desirable to have companion studies of the work in arithmetic and geometry prior to 1800 in our schools, but Doctor Simons' work shows an amount of research that would make a person hesitate to undertake these other tasks.

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